

Analysis of the Effects of Failure and Noise in the Distributed Connectivity Maintenance of a Multi-robot System

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Abstract—To perform cooperative tasks in a decentralized manner, multi-robot systems are often required to communicate with each other. Therefore, maintaining the communication graph connectivity is a fundamental issue when roaming a territory with obstacles. However, when dealing with real-robot systems, several sources of data corruption can appear in the agent interaction. In this paper, the effects of failure and noise in the communication between agents are analyzed upon a connectivity maintenance control strategy. The results show that the connectivity strategy is resilient to the negative effects of such disturbances under realistic settings that consider a bandwidth limit for the control effort. This opens the perspective of applying the connectivity maintenance strategy in adaptive schemes that consider, for instance, autonomous adaptation to constraints other than connectivity itself, e.g. communication efficiency and energy harvesting.

I. INTRODUCTION

In this paper, the effects of different disturbance types (communication failures and noise) upon the control strategy introduced in [1] are analyzed and its impact upon the connectivity maintenance of multi-robot systems is evaluated. As this is a decentralized strategy that requires local communication between agents, the main requirement is to keep the agents always connected during communication.

The strategy considered here is a representative of the so-called *global* connectivity techniques, which aim at maintaining a path between any pair of nodes (*i.e.* robots), with a possible elimination of redundant links and creation of new links as needed. This is in contrast with *local* connectivity techniques such as [2], [3] and [4], that guarantee that once a communication link between a pair of agents is active at time $t = 0$, it will be active $\forall t > 0$, thus establishing an initial connection path from the outset. From the Engineering point of view, however, imposing the maintenance of each single communication link is very costly, and a more intelligent scheme that removes redundant links and generate additional links as required can be more convenient. Indeed, information exchange among all the agents is guaranteed under a *global* connectivity of the communication graph. For example, in [5]

the idea of periodic connectivity, in which the network regains connectivity at predefined intervals, is introduced.

The control strategy introduced in [1] ensures that *global* connectivity is maintained in a system of agents. This approach is inspired in the strategy previously defined in [6] and introduces a way of obtaining, in a distributed way, the value of the second smallest eigenvalue of the Laplacian matrix, that, as shown in [7], measures the connectivity of a graph. Regarding other estimate procedures that can be found in [8] and [9], one of the principal advantages of the method described in [1] is that it provides, besides the Fiedler eigenvalue, estimations of its own gradients that are useful in real-time computations, as will be verified below.

II. BACKGROUND ON GRAPH THEORY

The connectivity scheme considered here is based on graph-theoretical considerations that are outlined in this section. Further background details can be found in [10].

Given N mobile robots, the instantaneous communication links among them are modeled as edges in an undirected graph where each robot corresponds to a node. Communication is assumed to be local, in the sense that each robot communicates only with a topological neighbourhood formed by nearby robots. Thus, \mathcal{N}_i is defined as the neighborhood of the i -th robot, *i.e.* the set of robots that can exchange information with it. The complete communication graph is represented by the adjacency matrix $A \in \mathbb{R}^{N \times N}$, where each a_{ij} is defined as the weight of the edge between robots i and j , and is positive if $j \in \mathcal{N}_i$, zero otherwise. This value needs to be computed in order to allow the local Laplacian evaluation, as we'll see next. As undirected graphs are being considered, $a_{ij} = a_{ji}$.

Finally, consider the Laplacian matrix of the graph, defined as $L = D - A$ where $D = \text{diag}(\{d_i\})$, and $d_i = \sum_{j=1}^N a_{ij}$ is the degree of the i -th node of the graph. L holds some important properties, among which a remarkable relationship between its eigenvalues and the graph connectivity. Namely, let λ_i , $i = 1, \dots, N$ be the eigenvalues of L . Then

- The eigenvalues can be ordered such that

$$0 = \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N \quad (1)$$

- $\lambda_2 > 0$ if and only if the graph is connected: λ_2 is then defined as the algebraic connectivity of the graph.

This means that any procedure that keeps the second eigenvalue λ_2 of the communication graph at positive values guarantees graph connectivity, *i.e.*, guarantees a communication path between any pair of nodes.

III. CONNECTIVITY MAINTENANCE

In this section a control strategy assuming that each agent can compute the actual value of λ_2 is first summarized. This hypothesis will be removed in the sequence, with the description of the distributed procedure.

A. Centralized Connectivity Maintenance Control Strategy

For the sake of clarity, a brief synopsis of the estimation procedure introduced in [6] will now be made. The estimation of λ_2 is computed by each agent through an estimate of the corresponding eigenvector v_2 .

Considering a group of N single integrator agents, let $p_i \in \mathbb{R}^m$ be the state vector describing the position of the i -th agent and u_i be the its control input. Then:

$$\dot{p}_i = u_i^c \quad (2)$$

where $p = [p_1^T \dots p_N^T]$ and u_i^c is defined as:

$$u_i^c = csch^2(\lambda_2 - \epsilon) \cdot \frac{\partial \lambda_2}{\partial p_i} \quad (3)$$

This control law ensures connectivity in a centralized framework, supposing each agent can compute the real value of the algebraic connectivity of the system. Subsequently, this assertion will be removed with the decentralized estimation procedure in Section III-B.

The maximum communication range for each agent is denoted by R , and the j -th agent is inside \mathcal{N}_i if $\|p_i - p_j\| \leq R$. In order to implement the estimation procedure in a decentralized manner, a brief overview of the estimation procedure proposed by [1] is presented in the next subsection. Specifically, the estimation of λ_2 is computed through the estimation of the corresponding eigenvector v_2 .

According to [6], λ_2 can be computed in a centralized framework as:

$$\frac{\partial \lambda_2}{\partial p_i} = \sum_{j \in \mathcal{N}_i} -a_{ij} \left(v_2^i - v_2^j \right)^2 \cdot \frac{p_i - p_j}{\sigma^2} \quad (4)$$

where the edge-weights a_{ij} are defined as in Eq. (5):

$$a_{ij} = \begin{cases} e^{-((\|p_i - p_j\|)^2)/(2 \cdot \sigma^2)} & \text{if } \|p_i - p_j\| \leq R \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where the scalar parameter σ is chosen to satisfy the boundary condition $e^{-(R^2)/(2 \cdot \sigma^2)} = \Delta$, in which Δ is a small defined threshold.

B. Decentralized Connectivity Maintenance Control Strategy

The control law implemented in this work was proposed in [1] and extends the one introduced in Eq. (2) by adding a bounded control term u^d to obtain some desired behavior:

$$\dot{p}_i = u_i^c + u_i^d \quad (6)$$

where u_i^c is given by Eq. (7):

$$u_i^c = csch^2(\lambda_2 - \epsilon) \cdot \frac{\partial \tilde{\lambda}_2}{\partial p_i} \quad (7)$$

To implement the strategy described in [11] in a decentralized way, each agent must compute an estimate of a component of the eigenvector v_2 , given by \tilde{v}_2^i . Let $\tilde{v}_2 = [\tilde{v}_2^1 \dots \tilde{v}_2^N]^T$ and $\tilde{\lambda}_2$ be the second smallest eigenvalue that the Laplacian matrix would take if \tilde{v}_2 were the corresponding eigenvector. So $\tilde{\lambda}_2$ can be computed in the following way:

$$\tilde{\lambda}_2 = \frac{k_3}{k_2} \cdot [1 - Ave((\tilde{v}_2^i)^2)] \quad (8)$$

According to the procedure given in [11], each agent can compute its own estimate of λ_2 using:

$$\frac{\partial \tilde{\lambda}_2}{\partial p_i} = f(\tilde{v}_2^i, \tilde{v}_2^j) = \sum_{j \in \mathcal{N}_i} -a_{ij} \left(\tilde{v}_2^i - \tilde{v}_2^j \right)^2 \cdot \frac{p_i - p_j}{\sigma^2} \quad (9)$$

However, the actual value of $\tilde{\lambda}_2$ cannot be computed by each agent, because actually, the real value of $Ave(\{(\tilde{v}_2^i)^2\})$ cannot be calculated in a distributed way. Nevertheless, an estimate of this average is available to each agent (see [6] for further details).

As shown in [11], λ_2^i is a good estimate of both λ_2 and $\tilde{\lambda}_2$. In other words, it has been shown that, given a positive Ξ value, it is always possible to guarantee that the absolute difference between the real (and distributedly unreachable) value assumed by λ_2 and its estimate obtained by agent i (given by λ_2^i) is bounded from 0 by Ξ for every agent. Additionally, it has also been shown that, given another positive Ξ' value, it is always possible to ensure that the absolute difference between the second smallest eigenvalue of L (that can be computed distributedly and denoted by $\tilde{\lambda}_2$) and its estimate obtained by agent i (given by λ_2^i) is bounded from 0 by Ξ' .

That is, it can be concluded then that, given a positive integer Ξ'' (given by $\Xi + \Xi'$), the absolute difference between the real (and distributedly unreachable) value assumed by λ_2 and the second smallest eigenvalue of L (which can be computed distributedly) is bounded from 0 by Ξ'' . Put differently, each agent is able to (locally) compute using $\tilde{\lambda}_2$ instead of λ_2 and still be able to obtain a valid measure of the (global) system connectivity.

Even though the actual value of $\tilde{\lambda}_2$ is not available to each agent, it is worth noting that the partial derivatives of $\tilde{\lambda}_2$ can be implemented in a decentralized manner. This way, each robot can locally estimate the value of λ_2 using Eq. (8).

1) *Rendezvous and Formation Control Strategy*: This section will summarize the main notions of the control strategy studied in this work to address the connectivity maintenance within rendezvous and formation control problems.

The edge-weights \bar{a}_{ij} were redefined so that the connectivity control action was implemented in its vector form:

$$\bar{a}_{ij}(\lambda_2^i) = \gamma_i \cdot \text{csch}^2(\lambda_2^i - \bar{\epsilon}) \cdot \frac{1}{\sigma^2} \cdot (\tilde{v}_2^i - \tilde{v}_2^j)^2 \cdot a_{ij} \quad (10)$$

Additionally, a modified degree matrix of the graph was introduced as $\bar{D} = \text{diag}(\{\bar{d}_i\})$, where $\bar{d}_i = \sum_{j=1}^N \bar{a}_{ij}$, i.e. represents the degree of the i -th node of the graph. The modified laplacian matrix of the graph, then, is redefined accordingly as $\bar{L} = \bar{D} - \bar{A}$, and the control law of the system assumes the form:

$$\dot{p} = -\bar{L} \cdot p + u^d \quad (11)$$

where u^d is a vector containing the control laws for each one of the N agents.

Consensus-Based Rendezvous: According to [12], the following consensus-based control law guarantees the convergence of the robots to the same position if the communication graph is connected:

$$u^d = -L * p \quad (12)$$

Consensus-Based Formation Control: This is the final form of the implemented algorithm. Instead of migrating to the same position, the agents assume a specific formation. As stated in [13], the following control strategy can be used, by adding a bias term $b_i(p)$ to the control law in Eq. (12):

$$u^d = -L * p + b_i(p) \quad (13)$$

where

$$b_i(p) = \begin{cases} \sum_{j \in N_i} (1 + \bar{a}_{ij}(\lambda_2^i)) \cdot (\bar{p}_i - \bar{p}_j) & , \text{ if } \lambda_2^i > k \cdot \bar{\epsilon} \\ \sum_{j \in N_i} (1 + \bar{a}_{ij}(k \cdot \bar{\epsilon})) \cdot (\bar{p}_i - \bar{p}_j) & , \text{ otherwise} \end{cases} \quad (14)$$

for some $k > 1$, where \bar{p}_i represents the desired relative position for robot i in the formation. This way, when the estimate of the algebraic connectivity is sufficiently greater than $\bar{\epsilon}$ (i.e. $\lambda_2^i > k \cdot \bar{\epsilon}$), the bias term is computed with Laplacian matrix $\tilde{L} = \bar{L} + L_*$. When the value of the estimate of the algebraic connectivity falls and approaches $\bar{\epsilon}$, the bias term of the second case in Eq. (14) ensures u^d is bounded and guarantees the connectivity maintenance among the agents.

IV. THE DISTURBANCE MODEL

Each agent i computes its own v_2 estimate, given by v_2^i , and from it obtains the spatial variation of the corresponding eigenvalue, given by $\partial \tilde{\lambda}_2 / \partial p_i$ as explained in Section III-B. This estimation system can be represented by the block diagram in Fig. 1, where $f(\cdot)$ is given by Eq. (9).

In order to maintain the communication path between every pair of robots, each robot should be able to estimate

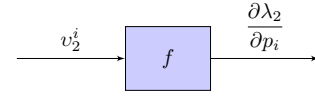


Fig. 1. Block Diagram of the Original Model

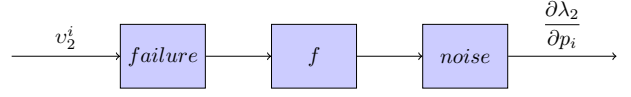


Fig. 2. Block Diagram of the Model with Disturbances

effectively the global connectivity of the system only with local information. However, both communication failures and data corruption (noise) coexist in a realistic multi-robot communication system that is expected to keep itself connected, even in the presence of such disturbances.

The model proposed in this work adds *failure* and *noise* as sources of corruption into the arrangement proposed in [11] to study its effects on the final system performance. This situation can then be represented by the block diagram in Fig. 2.

A. Modelling failures

The local estimation of v_2 made by each agent is a critical process to keep the system connected. Errors in this estimate automatically imply incorrect values of $\tilde{\lambda}_2$, which may result in $\lambda_2 = 0$. Given N agents interacting in an unknown environment, failures in data reception are considered. Thus, let $v_2^{i'}$ be the v_2 estimate made by agent i in the presence of a communication failure. In this case, it is assumed that the agent stipulates a default value $v_2^* = 1$ for the missing estimate:

$$v_2^{i'} = \begin{cases} v_2^* & , \text{ for } p = p_{fail} \\ v_2^i & , \text{ for } p = 1 - p_{fail} \end{cases} \quad (15)$$

where p_{fail} is the failure probability.

B. Modelling noise

The model adopted to simulate the effects of noise in the agents communication was the Additive White Gaussian Noise (AWGN). In real system simulations, noise is usually introduced in the communication channel modifying, in our case, the estimates of v_2 received from other agents. We model this process considering a noiseless communication channel that transmits an estimate already corrupted by noise when it enters it. Thus, let $v_2^{i'}$ be the v_2 estimate made by agent i in the presence of an independent and additive distributed noise Z_i . Thus:

$$v_2^{i'} = v_2^i + Z_i \quad (16)$$

where $Z_i \sim N(0, \eta)$ (Z_i represents the *gaussian noise*, that follows a normal distribution with $\mu = 0$ and $\sigma^2 = \eta$).

C. Control Strategy in the Presence of Disturbances

Let us consider a group of N single integrator agents, whose dynamics is described by Eqs. (6), (7) and (13). The v_2 estimate calculated by agent i , in the presence of communication noise and failures can be expressed by:

$$v_2^{i'} = \begin{cases} v_2^* + Z_i & , \text{ for } p = p_{fail} \\ v_2^i + Z_i & , \text{ for } p = 1 - p_{fail} \end{cases} \quad (17)$$

Consider λ_2' as the algebraic connectivity of a graph in the presence of disturbances and $\lambda_2^{i'}$ as the estimate of λ_2' made by agent i . If the disturbance effect considered is assumed to be limited, (that is, there is no failure in *any* communication between two nodes and noise is not *infinite*), it can be concluded then that, similarly to what was seen in the absence of disturbances, λ_2^i is a good estimate of both λ_2' and $\tilde{\lambda}_2'$.

It can be noticed that the estimation error of λ_2' (the connectivity measure in the presence of failure and/or noise) is limited (that is, $\exists \Xi > 0$ such that $|\lambda_2^{i'} - \lambda_2| \leq \Xi, \forall i = 1, \dots, N$). Likewise, it can be found that the difference between the connectivity estimates and the second smallest eigenvalue of L is also limited (in other words, $\exists \Xi' > 0$ such that $|\lambda_2^{i'} - \tilde{\lambda}_2| \leq \Xi', \forall i = 1, \dots, N$). Hence, λ_2' can be used to locally estimate the real value of λ_2 , inaccessible to the agents.

It is worth reminding that, besides not being possible to ensure that the estimation error of λ_2' tends to 0, its value is larger as the effect of one or more types of disturbances in the communication between agents gets larger.

V. MATLAB SIMULATIONS

The results of the main simulations and experiments presented in this work are available *online*¹.

A real-world robot system, both terrestrial (according to [14]) or aquatic (as in [15]), is expected to have a control effort acting on a frequency approximately in the order of 10 *rad/s*. Therefore, a *first-order low pass filter* was added to the control signal with a transfer function given by Eq. (18), attenuating the contribution of frequencies higher than 10 *rad/s* in u^c in order to predict the performance of the proposed strategy in physical robots of the real world.

$$H(s) = \frac{10}{s + 10} \quad (18)$$

A formation control problem with a varying number of agents ranging from $N = 3$ to $N = 10$ in an environment with $N_{obst} = 150$ obstacles was considered for our experiments. Simulations have been carried out by considering the following parameter sets: $p_{fail} = \{0, 0.05, \dots, 0.70\}$, $\eta = \{0, 0.1, 0.3, 0.5, 1.0, 5.0\}$.

For clarity reasons, the connectivity measure, its estimates and the control effort in the presence of disturbances (represented until now as λ_2' , $\lambda_2^{i'}$ and $u^{c'}$) will only be referred to as λ_2 , λ_2^i and u^c . The estimate of a component of v_2 made by agent i (denoted until now by \tilde{v}_2^i) will also be reported

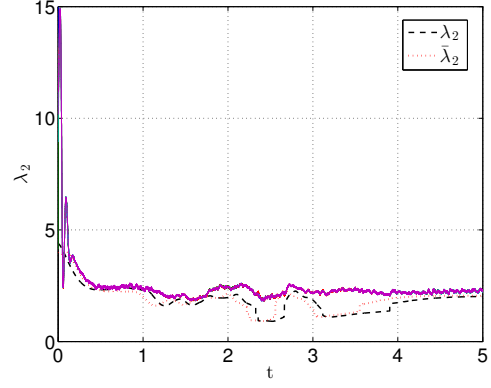


Fig. 3. λ_2 , λ_2^i and $\bar{\lambda}_2$ for $p_{fail} = 0.2$ and $\eta = 0$ with $N_{obst} = 150$

to simply as v_2^i and for reference purposes, the connectivity measure in the absence of disturbances will be represented as $\bar{\lambda}_2$.

The results presented consider a typical execution of the connectivity maintenance algorithm described in [11]. Typical runs of five agents performing formation control correspond to the robots starting at random initial positions and supposed to converge to a pentagonal configuration, while deviating from randomly placed point obstacles along the path.

A. Failure

In Figure 3, the connectivity and its estimates evolution for $N = 5$ agents interacting with $p_{fail} = 0.20$ on an environment with $N_{obst} = 150$ obstacles is shown.

As can be seen, even in the presence of failures in the communication process, the system kept itself connected. The initial estimates ($\lambda_2^i \approx 10.17$ in $t = 0$) reach a maximum at the beginning ($\lambda_2^i \approx 14.9$ in $t \approx 0.02$ s) of the dynamics, while the connectivity decreases (from $\lambda_2 \approx 4.40$ in $t = 0$) and temporarily stabilizes at a specific level ($\lambda_2 \approx 2.32$ from $t_1 \approx 0.44$ s till $t_2 \approx 1.07$ s), declining slowly as agents position themselves. The value of λ_2 changes because of the obstacle deviation process and varies from $\lambda_2 \approx 2.27$ in $t_{obst_0} \approx 1.07$ s, when agents start crossing the obstacles set, to $\lambda_2 \approx 1.68$ in $t_{obst_1} \approx 3.91$ s, when they leave the obstacle region, reaching a minimum of $\lambda_2^{min} \approx 0.91$ in $t_{min} \approx 2.53$ s. The simulation ends with $\lambda_2 \approx 2.01$ and $\lambda_2^i \approx 2.26$ in $t = 5$ s.

As can be observed, adding failures to the communication process did not modify substantially the results observed in [11] regarding the connectivity dynamics, because both λ_2 and λ_2^i kept themselves positive in the agent interaction. Compared to the results without any disturbances ($\bar{\lambda}_2$ in Fig. 3), because of $p_{fail} > 0$, the values of λ_2^i slightly deteriorate, but remain positive until the end of the simulation for $p_{fail} = 0.20$.

If the pseudorandom number generation (produced by the simulation environment) is approximated to a random one, then the failure probability is independent and equally distributed for *all* agents. Consequently, the *system failure* probability can be expressed by Eq. (19).

$$p_{fail}^{sys} = N \cdot p_{fail} \quad (19)$$

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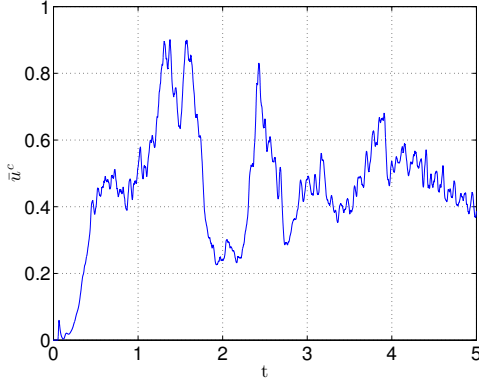


Fig. 4. u^c for $p_{fail} = 0.2$ and $\eta = 0$ with $N_{obst} = 150$

In this case, $p_{fail}^{sys} = 5 \cdot 0.2 = 1$ fail/interaction. That is, even with one robot failing at each interaction, on average the results observed in Fig. 3 show that the model responds well to this type of disturbance.

The evolution of u^c regarding this dynamics can be seen in Figure ?? In Fig. 3, it can be noticed that λ_2^i begins by stipulating high values for the system connectivity, and then u^c has low initial values. As the estimates diminish, u^c rises and agents start crossing the obstacles region (which is done between $t_{obst_0} \approx 1.07$ s and $t_{obst_1} \approx 3.90$ s), with a control effort variation between $u^{c_{min}} \approx 0.23$ in $t_1 \approx 1.91$ s and $u^{c_{max}} \approx 0.90$ in $t_2 \approx 1.38$ s.

For $p_{fail} = 0.30$ (not shown here), results were qualitatively similar to the examined case, with connectivity maintenance for typically-real controller agents. However, for $p_{fail} = 0.40$ connectivity was not maintained. In this situation, after reaching its maximum value, the estimates λ_2^i fell and ended up becoming negative for some time, which made λ_2 (that was positive) decline and end up becoming zero in the sequence. Once the connectivity among the agents was lost, each one of them get incorrect values for the position of its peers outside its communication range. In this case, the estimates λ_2^i lost their validity.

B. Noise

In Figure 5, the values of the connectivity and its estimates are represented for agents communicating with $\eta = 0.5$ on an environment with $N_{obst} = 150$ obstacles.

As can be seen, agents kept themselves connected even with noise in the communication process. The connectivity ($\lambda_2 \approx 4.00$ in $t = 0$) initially decreases and then grows until reaching its maximum ($\lambda_2 \approx 4.52$ in $t \approx 0.13$ s), while the estimates range between $\lambda_2^{i_{min}} \approx 0$ and $\lambda_2^{i_{max}} \approx 9.05$. The connectivity varies from $\lambda_2 \approx 3.32$ in $t_{obst_0} \approx 1.04$ s (when agents start going through the obstacles region) until $\lambda_2 \approx 3.88$ in $t_{obst_1} \approx 3.63$ s (when they finish crossing it), with noticeable variations caused by the noisy connectivity estimates. Under the influence of noise, the connectivity maintenance control action is impaired and does not stabilize at a specific value at the end of dynamics, as can be observed in

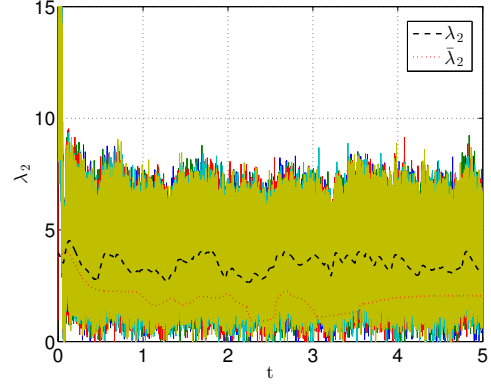


Fig. 5. λ_2 and λ_2^i for $\eta = 0.5$ and $p_{fail} = 0$ with $N_{obst} = 150$

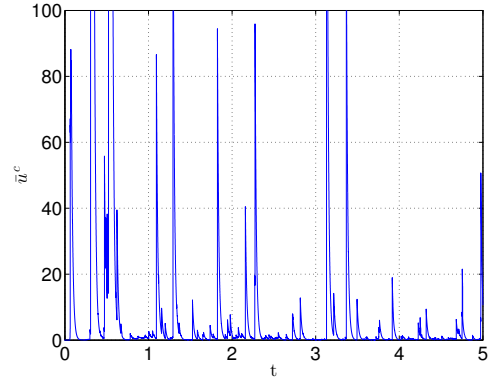


Fig. 6. u^c for $\eta = 0.5$ and $p_{fail} = 0$ with $N_{obst} = 150$

the value of λ_2 in Figure 5.

As noise is uniformly distributed, its influence on λ_2^i ends up “masking” the agents temporary separation movement (and the consequent λ_2 descents) to the connectivity maintenance algorithm as the obstacles region is crossed. Therefore, λ_2 varies mostly as a consequence of noise throughout the interaction, and does not show any significant drop given by the obstacle deviation process (there is no valley in λ_2 between t_{obst_0} and t_{obst_1}). Dynamic ends with $\lambda_2 \approx 3.12$ and $0.62 < \lambda_2^i < 6.89$ in $t = 5$ s.

Thus, communication noise did not affect substantially the connectivity dynamics previously obtained for the ideal-controller agents with no disturbances observed in [11], despite harming the estimation procedure. Actually, agents just do not take exactly their own formation position during the dynamics, but oscillate around it, instead.

The evolution of u^c regarding this dynamics can be found in Figure 6.

In the beginning, a local maximum in the total control effort ($u^c \approx 88.2$ in $t \approx 0.07$ s) can be observed, followed by the global maximum ($u^c \approx 3.069$ in $t \approx 0.52$ s, in a not shown measure) as agents enter formation. Due to noise, agents poorly identify their own state and, as dynamic starts, they happen to reapproximate instead of taking their own formation position (as can be seen on the simulation videos referenced

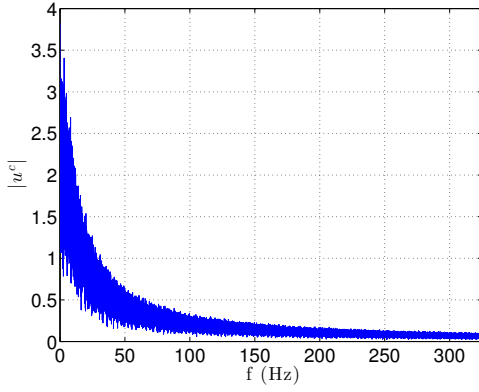


Fig. 7. Frequency spectrum of u^c for $\eta = 0.5$ and $p_{fail} = 0$

in the footnote) which explains the high value assumed by u^c to put them in formation. Following that, the obstacles region start being traversed (between t_{obst_0} and t_{obst_1}) and the value of u^c decreases in intensity, because of the separation movement imposed by the obstacle avoidance action, but still demands local maxima in u^c (as $u^c \approx 275$ in $t \approx 3.13$ s, for example) in order to keep the connectivity maintenance. Finally, after passing the obstacles, values assumed by u^c decrease in intensity (with a local maximum of $u^c \approx 50.4$ in $t \approx 4.98$ s) but end dynamics taking local maxima to avoid disconnection in the presence of noise.

An analysis of the u^c spectrum (see Fig. 7) shows that a system composed by typically-real agents demands, for the connectivity maintenance in the presence of a noise rate $\eta > 0$, a control effort with an increased participation in high frequencies. In this case, these components are responsible for keeping the small amplitude variation around λ_2 during the obstacles deviation in Fig. 5. As $\eta = 0.5$, a noisy signal $Z_i \sim N(0, 0.5)$ is added to the original information, what makes the values assumed by v_2^i distribute themselves according to a normal distribution. As u^c acts on a signal that changes more frequently (as consequence of noise), the presence of high frequency components in u^c increases, as can be noted in Fig. 7 for $f > 10$ Hz.

VI. CONCLUSION

In this work, an analysis of the effect of different types of disturbances over a control algorithm was made, which through a decentralized estimation of the algebraic connectivity in a communication graph, guarantees the connectivity maintenance in a multi-robot system, given a random initial setup.

In the given analysis, the *group connectivity was always kept for ideal agents* (in which there is an equitable response both for high and low frequencies of the control effort, in results not shown). As can be seen in Section V, the high-frequencies performance can have an important role in guaranteeing the connectivity maintenance between typically-real agents in the presence of high-levels of disturbance. In that case, analyses can be summarized as follows:

The model is failure-tolerant until a high specific p_{fail}^{max} probability (where $p_{fail}^{max} < 40\%$). From this value, the v_2 -estimates made by agents are not sufficient to ensure $\lambda_2 > 0$ while they move through a region with randomly-placed point obstacles.

The model is noise-tolerant to every of the $\eta > 0$ cases analyzed, once this type of disturbance does not completely deteriorate the value of v_2^i used in the estimation process of λ_2^i . Thereby, it is always possible for each agent, even though inaccurately, to compute u^c in order to keep $\lambda_2 > 0$ if $\eta > 0$.

To put it briefly, as the connectivity maintenance is a necessary condition to the estimation procedure described in [11], once it is lost, it is not possible to rely on the local estimates (λ_2^i) done by agents in the presence of high failure rates. However, in the real world, agents have inertial characteristics that prevent quick disconnections, and usually it is possible to recover from such situations by resetting the operation of the algorithm, for example.

Current work aims at proposing a strategy to smooth the disturbances impact in the global connectivity maintenance of a multi-robot system through the use of a Linear-Quadratic Estimator, filtering the communication noise in v_2^i or decreasing the influence of communication failures in the agent interaction.

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